Lecture 07
Warehousing Design and Design of Fast-Pick Area

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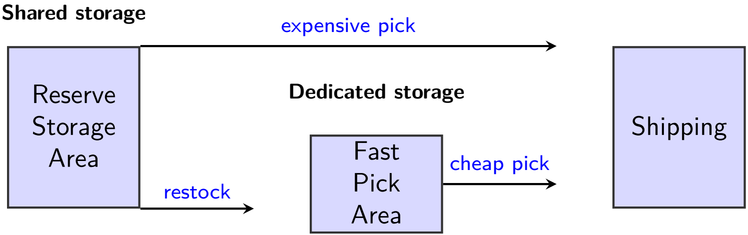
Outline

source: General references [BH09, Mul94, Fra02, **?**]

What is a fast-pick area?

- **What :** compact & picking efficiently warehouse
- **Idea:** moving 'small'& 'popular' SKUs to gain efficiency
- **Pro:** reduce searching & traveling time
- **Con:** double handling, additional refilling, additional equipment
- **Example:** Avon cosmetic, Office Depot ink carriages, Gums shelve casher

Concept of Fast-Pick Area



source: Bartholdi, J. & Hackman, S. 2009. [BH09]

Questions for Fast-Pick Area

1) What are the optimal total spaces of the fast-pick area?
2) Which SKUs should be put in the fast-pick area?
3) How much space each SKU should be allocated?

Trade-off between additional restocking & inexpensive picking with **fluid model** [HP90]

Assumptions

- Each piece is very 'small'
- Restocking cost depends on # of restock (c_r)

How much space for each SKU?

Notations

$$\begin{aligned}
\mathcal{I} &= \text{set of SKUs in warehouse} \\
\mathcal{F} &= \text{set of SKUs in fast pick area, } \mathcal{F} \subseteq \mathcal{I} \\
f_i &= \text{flow of SKU } i \text{ within one period of time } (m^3/\text{time}) \\
v_i &= \text{assigned volume of SKU } i \text{ in fast-pick area } (m^3) \\
V &= \text{total available volume in the fast-pick area } (m^3)
\end{aligned}$$

- **Decision variable:** v_i
- **Constraints:** $\sum_i v_i \leq V$
- **Objective:** min restocking cost = min # restocking
- **# restock:** $\frac{\text{flow}}{\text{assigned volume}}$ or $\frac{f_i}{v_i}$ for SKU i

Three popular policies

Assuming that $\mathcal{F}$ is given, then total restock costs:

$$\min \ z = \ c_r \sum_{i \in \ \mathcal{F}} \frac{f_i}{v_i}$$

s.t.

$$\sum_{i \in \ \mathcal{F}} v_i \qquad \leq \ V$$

Three popular policies

- **Equal space:** each SKU is assigned equal space
- **Equal frequency:** each SKU will be refilled equal frequency
- **Optimal:** ???

Examples

The annual sale quantities of SKUs A & B are 90 & $160m^3$, respectively. What are numbers of restocks of each SKU, if total volume in FPA is 50 m^3

A) each SKU get $25m^3$

B) SKUs A & B get 18 & $32m^3$, respectively

C) SKUs A & B get 21.43 & $28.37m^3$, respectively

	A) space	B) frequency	C) customize
space to SKU (v_A, v_B)	$(25, 25)$	$(18, 32)$	$(21.43, 28.37)$
restock to SKU	$(3.6, 6.4)$	$(5.0, 5.0)$	$(4.2, 5.6)$
total restock	10	10	9.8

- \# restock of equal space = # restock equal frequency
- How to compute optimal policy (i.e., customize)

Why equal space = equal time?

- **What:** each SKU is assigned equal space in fast-pick area
- **Space:** $v_1 = v_2 = \ldots = v_n \rightarrow \ \frac{V}{n}$
- **# Restock:** $\frac{f_i}{v_i} = \frac{n\,f_i}{V}, \forall\ i \in \mathcal{F}$
- **Total restocks:** $\sum_{i\in\ \mathcal{F}} \frac{n\,f_i}{V} = \frac{n}{V} \sum_{i\in\ \mathcal{F}} f_i$

- **What:** each SKU will be refilled equal time/frequency in fast-pick area
- **Space:** $v_i = \frac{f_i}{f_1} v_1 \quad V = \sum v_i = \frac{v_1}{f_1} \sum f_i$
- **# Restock:** $\frac{f_1}{v_1} = \frac{f_2}{v_2} = \ldots = \frac{f_n}{v_n} = \alpha$

  $v_i = \frac{f_i}{\alpha}$ Hence, $\sum_{i\in\ \mathcal{F}} v_i = \sum_{i\in\ \mathcal{F}} \frac{f_i}{\alpha} = V$
- **Total restocks:** $\sum_{i\in\ \mathcal{F}} \frac{f_i}{v_i} = n\alpha = \frac{n}{V} \sum_{i\in\ \mathcal{F}} f_i$

Optimal space ratio $= \frac{\sqrt{\mathrm{FLOW}_i}}{\sum_{j \in \mathcal{F}} \sqrt{\mathrm{FLOW}_j}}$

$$
\begin{aligned}
\min z &= C_r \left(\frac{f_a}{v_a} + \frac{f_b}{v_b} \right) \\
\text{s.t.} & \\
v_a + v_b &\geq V
\end{aligned}
$$

Set constraint binding

$$C_r \left(\frac{f_a}{v_a} + \frac{f_b}{V - v_a} \right)$$

Apply FOC, i.e., $\frac{\partial}{\partial v_a} z = 0$

$$
\begin{aligned}
C_r \left(-\frac{f_a}{v_a^2} + \frac{f_b}{(V - v_a)^2} \right) &= 0 \\
\frac{f_a}{v_a^2} &= \frac{f_b}{(V - v_a)^2} \\
v_b &= \frac{\sqrt{f_b}}{\sqrt{a}} v_a = \frac{\sqrt{f_b}}{\sqrt{a} + \sqrt{f_b}} V
\end{aligned}
$$

Summary

	Equal time	Equal space	Optimal
space to SKU i	$\frac{f_i}{\sum_{j \in \mathcal{F}} f_j} V$	V/n	$\frac{\sqrt{f_i}}{\sum_{j \in \mathcal{F}} \sqrt{f_j}} V$
restock to SKU i	$\frac{1}{V} \sum_{j \in \mathcal{F}} f_j$	$\frac{n f_i}{V}$	$\frac{\sqrt{f_i}}{V} \sum_{j \in \mathcal{F}} \sqrt{f_j}$
total restock	$\frac{n}{V} \sum_{j \in \mathcal{F}} f_j$	$\frac{n}{V} \sum_{j \in \mathcal{F}} f_j$	$\frac{1}{V} (\sum_{j \in \mathcal{F}} \sqrt{f_j})^2$

Example 2: Space for each SKU

A warehouse manager considers to put three items, i.e., SKUs A, B, & C, into a newly develop fast-pick area. The historical data of these items are listed as follow:

SKU	flow (f_i)
A	$4m^3$ per year
B	$2m^3$ per year
C	$1m^3$ per year

If the total storage space of fast-pick area is $0.6m^3$, what are space & number of restock of each SKU if he implements (a) optimal policy, (b) equal time policy, & (c) equal space policy.

Solutions to Example 2

Space

	Equal time	Equal space	Optimal
SKU A	$\frac{4}{4+2+1}0.6$	0.2	$\frac{\sqrt{4}}{\sqrt{4}+\sqrt{2}+\sqrt{1}}0.6$
SKU B	$\frac{2}{4+2+1}0.6$	0.2	$\frac{\sqrt{2}}{\sqrt{4}+\sqrt{2}+\sqrt{1}}0.6$
SKU C	$\frac{1}{4+2+1}0.6$	0.2	$\frac{\sqrt{1}}{\sqrt{4}+\sqrt{2}+\sqrt{1}}0.6$

Restock

	Equal time	Equal space	Optimal
SKU A	11.66	20.0	14.71
SKU B	11.66	10.0	10.42
SKU C	11.66	5.0	7.35
Total	34.98	35.0	31.48

Which SKUs in fast-pick area?

- **Possible SKU in fast pick:** $2^{|\mathcal{I}|}$ if $\mathcal{I}$ is set of all SKUs
- **Reality Check:** two extremes case of fast-pick area ($\mathcal{F}$)

  $\mathcal{F} = \emptyset$ imply no saving from fast-pick area

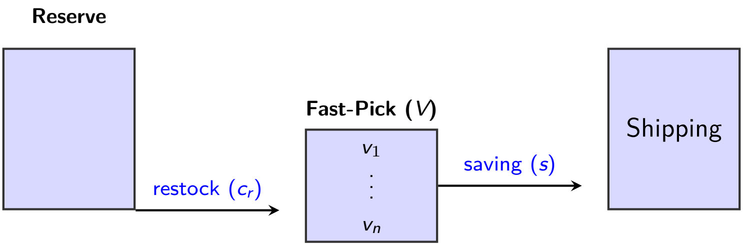
  $\mathcal{F} = \mathcal{I}$ too many activity (both pick & restocks)
- **Why 'small' & 'popular' SKUs?:**

  POPULAR generate high # pick

  SMALL reduce # restock

- benefit per pick = difference unit picking cost of fast pick & reserve
- For each SKU in fast pick area, benefits > restock costs

Fluid Model



source: Bartholdi, J. & Hackman, S. 2009. [BH09]

- **Total benefits:** $\sum$ (saving $\times$ pick of each SKU)
- **Total costs:** $\sum$ (restock cost $\times$ # restock of each SKU)

Fluid Model: Additional Notations

$$s = \text{pick saving when a SKU move to fast-pick area } (\$/\text{pick})$$
$$p_i = \text{picking rate of SKU } i \text{ (pick/time)}$$
$$c_r = \text{restocking cost at the fast-pick area } (\$/\text{restock})$$

- **Variable:** select set of SKUs
- **Constraint:** N/A
- **Objective:** $\max \sum$ [benefits - restock costs]

$$\max \sum_{i\in \mathcal{F}} \left[s\, p_i - c_r \frac{f_i}{v_i} \qquad \right]$$

Which SKU should be in fast pick area

$$\text{Total Net Benefit} \quad = \quad \sum_{i \in \mathcal{F}} \left[s\, p_i - c_r \frac{f_i}{v_i} \qquad \right]$$

Analysis: each SKU in fact pick, say k: Its benefit must be ≥ 0, say ϵ

$$\begin{aligned}
\text{Net Benefit}_k &= s\, p_k - c_r \frac{f_k}{v_k} \\
&= s\, p_k - c_r \frac{f_k}{\frac{\sqrt{f_k}}{\sum_{j \in \mathcal{F}} \sqrt{f_j}} V} \\
&= s\, p_k - c_r \frac{(\sqrt{f_k}) \cdot (\sum_{j \in \mathcal{F}} \sqrt{f_j})}{V} \quad = \epsilon
\end{aligned}$$

$$\frac{p_k}{\sqrt{f_k}} = c_r \frac{\sum_{j \in \mathcal{F}} \sqrt{f_j}}{s\, V} + \epsilon \frac{\sqrt{f_k}}{s}$$

Results from Fluid Model

selecting SKU to Fact-Picking area

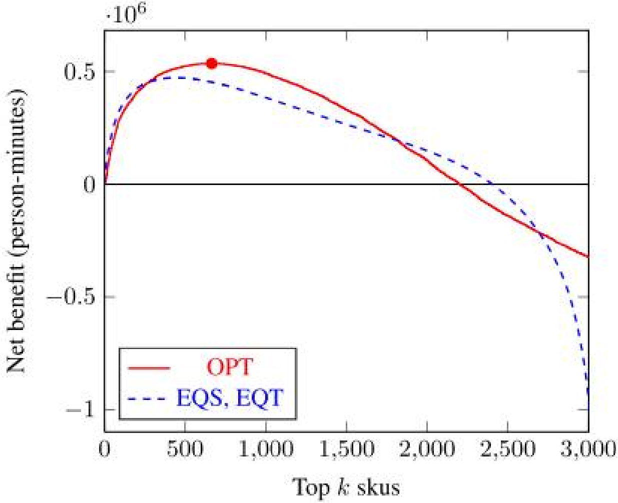
- Parameters are: $\sqrt{\text{flow}}$ & pick, particularly $\frac{\text{pick}}{\sqrt{\text{flow}}}$
- Term $\frac{\text{pick}}{\sqrt{\text{flow}}}$ is called viscosity

Hence

THRESHOLD SKU i is in fast-pick area if $\frac{p_i}{\sqrt{f_i}} > \frac{C_r}{s\,V} \sum_{j \in \mathcal{F}} \sqrt{f_j}$

PRACTICAL adding SKU in descending order of viscosity until total net benefit stops increases

Net Benefits Comparison



source: Bartholdi, J. & Hackman, S. 2009. [BH09]

Example 3: Which SKU in fast pick area

From the previous example, the historical pick data are follow:

SKU	pick (p_i)	flow (f_i)	viscosity ($\frac{p_i}{\sqrt{f_i}}$)
A	20 times	$4m^3$ per year	10.0
B	30 times	$2m^3$ per year	21.2
C	25 times	$1m^3$ per year	25.0
D	15 times	$2m^3$ per year	10.6

If each restock take 5 minutes & moving item into a fast-pick area takes traveling & searching time 2 minutes per each pick. As a logistics analysis, what is your recommendation to a warehouse manager choice of SKUs.

Example 3: Calculating Saving

Case 1: $\mathcal{F} = \{C\}$

SKU	v_i	saving $(s \cdot p_i)$	restock $(c_r \cdot \frac{f_i}{v_i})$	benefit (minutes)
C	0.600	(25)(3)	(5)(1)/(0.6)	66.67

Case 2: $\mathcal{F} = \{C, B\}$

SKU	v_i	saving $(s \cdot p_i)$	restock $(c_r \cdot \frac{f_i}{v_i})$	benefit (minutes)
C	0.250	(25)(3)	(5)(1)/(0.25)	55.00
B	0.350	(30)(3)	(5)(2)/(0.35)	61.43

Case 3: $\mathcal{F} = \{C, B, D\}$

SKU	v_i	saving $(s \cdot p_i)$	restock $(c_r \cdot \frac{f_i}{v_i})$	benefit (minutes)
C	0.156	(25)(3)	(5)(1)/(0.156)	42.95
B	0.222	(30)(3)	(5)(2)/(0.222)	44.95
D	0.222	(15)(3)	(5)(2)/(0.222)	-0.05

Example 3: Saving List

$\mathcal{F}$	$\sum_j \sqrt{f_j}$	$\mathbf{v}$	benefits (s)
$\emptyset$	0.00	0	0
$\{C\}$	1.00	$\{.6\}$	66.6
$\{C, B\}$	2.41	$\{.250, .350\}$	116.43
$\{C, B, D\}$	3.83	$\{.156, .222, .222\}$	87.85
$\{C, B, D, A\}$	5.83	$\{.104, .145, .145, .206\}$	

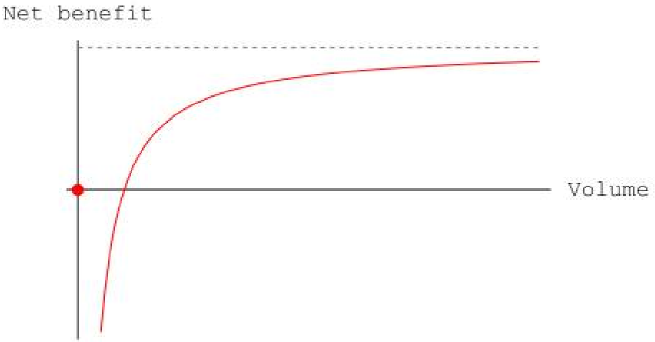
Factors related to decision

- **Warning:** 'fast-pick area' is not for every warehouse
- **Space:** large space $\rightarrow$ more SKU but less saving
- **Equipments:** benefits, capacity, investmet
- **Pickers:** management (fixed or circulated pickers), payment, restocker

Determine size of fast pick area

- **Method:** trial-and-error (smart search)
- **Rule of thumb:** fast pick space $< \frac{1}{4}$ of storage space
- **Issue:** different saving in each size/zone,

Net Benefits of Using Fast Pick Area



source: Bartholdi, J. & Hackman, S. 2009. [BH09]

Real World Fast-Pick Area

- **Discrete quantity:** Real SKU has shape & size, not fluid scale-able volume
- **Pick pattern:** #pick pattern $\nrightarrow$ uniform pattern nor equally distributed
- **Pick relationship:** Few SKUs must pick together, not independent pick
- **Equipment constraint:** multiple picking method, no fractional slot
- **Parameter:** different pick saving and/or restock cost [KM08]

Generalization: Improving Model

- **Minimum/Maximum quantity:** $\underline{v}_A \leq \ v_A \leq \ \overline{v}_A$

  if $\frac{\sqrt{f_A}}{\sum_j \sqrt{f_j}} V > \overline{v}_A$, then $v_A = \overline{v}_A$

  if $\frac{\sqrt{f_A}}{\sum_j \sqrt{f_j}} V < \underline{v}_A$, then $v_A = \underline{v}_A$

- **Affiliated picking:** SKUs A & B must be together $(p_A \ = \ p_b \ = \ p_{A,B})$

  calculate $\frac{p_{A,B}}{\sqrt{f_{A,B}}}$

- **Unit load: Fluid Model** breaks down

  0/1/All rule

Problems

Suppose you have 2 cubic meters available in flow rack, which is restocked from a distant reserve area, & you have just added three SKUs, with projected activity as follows.

SKU	picks/month	pieces/month	pieces/case	m^3/case
A	1000	2000	200	0.2
B	300	1200	6	0.7
C	250	4000	10	0.1

- Suppose you have decided to put all three SKUs in flow rack. How much space should be allocated to each SKU to minimize number of total restock?
- Based on the previous question, how often must each SKU be restocked?
- Assume that it costs an average of \$0.15 per pick from flow rack but costs about \$1/restock. The alternative is to pick from reserve, where each pick costs about \$0.25. Which SKUs should put in the flow rack? & How much space should they be allocated?

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