2104529 Computational Method for IE
Workshop 3: Unconstraint Optimization

Question 1

Consider the following function

$$f(\mathbf{x}) = (x_1 - 4)^4 + (x_2 - 3)^2 + 4(x_3 + 5)^4,$$

where $\mathbf{x} = [x_1, x_2, x_3]^T \in \mathbb{R}^3$ with $[4.0, 2.0, -1.0]^T$ as initial point.

(a) plot contour of this function around $x_1 = 4$, $x_2 = [0, 8]$, $x_3 = [-1, -9]$

(b) compute gradient vector and hessian matrix at the initial point

(c) apply Steepest Decending Code to solve this question steepestDescend.R

(d) solve the problem with R script using Descending Method using fixed stepsize

$$\gamma_k = \frac{2}{\lambda_{\max}(\nabla^2 f(\mathbf{x_k}))}, \text{ where } \lambda_{\max}(\nabla^2 f(\mathbf{x_k})) \text{ is eigenvalue}$$

for 5 iterations using and compare result with Steepest Descending Method in term of direction and stepsizes

(e) [0 points (bonus)] find the optimal solution using R command optim $(\cdot)$

(f) [0 points (bonus)] apply Newton's Method to solve the problem pureNewton.R

Question 2

Using Newton Method to find solution of this Powell function

$$f(\mathbf{x}) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$$

for three iteration with a initial points is $\mathbf{x_0} = [3, -1, 0, 1]^T$ under the following condition.

$$\gamma_k = \max\{\frac{1}{2^n} : f(\mathbf{x} - \frac{1}{2^n}\nabla f(\mathbf{x_k})) \leq \ f(\mathbf{x_k}), n \in \mathbb{Z}^+\}$$

(a) use a variation of Gradient Method in which γ_k as the stepsize in iteration k

(b) use a variation of Newton Method in which γ_k as the stepsize in iteration k

(c) compare two algorithms in terms of stepsize, search direction, quality of solution, and gap from optimal solution

(d) [0 points (bonus)] Using R command optim $(\cdot)$

Question 3

Consider the results of experiments

	1	2	3	4	5	6	7	8	9	10
t	0.1	1.2	2.1	4.6	5.6	5.9	6.2	6.8	8.1	9.3
y	1.816	0.267	-0.304	-0.849	-1.053	-1.200	-1.260	-1.294	-1.628	-1.881

If the best function to describe the relationship is:

$$\hat{y} \;=\; x_1 + x_2\, t + x_3\, e^{x_4 t}$$

(a) Formulate nonlinear function that minimize sum square error

(b) Solve the nonlinear function using Newton Method and the initial $\mathbf{x_0} = [0.1, -0.2, 2.0, 1.0]^T$

(c) Estimate parameters $(x_1,\ x_2, x_3, x_4)$ using Guass-Newton Method and the initial $\mathbf{x_0} = [0.1, -0.2, 2.0, 1.0]^T$

(d) find the optimal parameters $(x_1,\ x_2, x_3, x_4)$ using R command nls $(\cdot)$

Question 4

Consider facility location problem that minimize total transportation cost from a single warehouse X to/frome the following locations.

No.	customer	location ($\mathbf{p_i}$)	# shipments (w_i)
1	A	(8,2)	9
2	B	(3,10)	7
3	C	(8,15)	2
4	D	(3,4)	6
5	E	(16,8)	7
6	F	(4,5)	5
	X	(x, y)	-

(a) plot locations of each customer along its their shipments and guess the location of warehouse

(b) locate the warehouse that minimizes the total transportation cost that is estimated by

$$2\sum_i w_i \cdot d(p_i, X)$$

(c) plot contour of total weight distance in pervious questions and guess the solution

(d) locate the warehouse that minimizes the total transportation cost that is calculated by

$$\sum_i \left[w_i \cdot d(p_i, X) + \max(w_1^{1/2} \cdot d(p_i, X)^{3/2}, w_i^{3/2} \cdot d(p_i, X)^{1/2}) \right]$$

, locate the warehouse that minimizes the transportation cost

(e) [0 points (bonus)] repeat calculation if the location and weight of facilities are generated

where $d(p_1, p_2)$ is Euclidean distance between points p_1 and p_2

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